APPLECROSS
SENIOR HIGH SCHOOL

Name: ___SOLUTIONS___

		Total	
Section 1		27 28	
Section 2		23 22	**%**
Total		50	

All working is to be shown in the space provided. Your working should be in sufficient detail to allow your answers to be checked readily so part marks may be awarded if the answer is incorrect. For any question worth more than 2 marks valid working or justification must be shown to be awarded full marks.

SECTION 1 – Resource Free

Working Time: 30 minutes

1. **[5 marks]**

(a) Show clearly how the quadratic formula can be used to solve (3)

$$3x^2 – 5x – 1 = 0$$

leaving the answer in exact form.

$a = 3, \quad b = -5, \quad c = -1$

$b^2 - 4ac = 25 + 12$

$\qquad = 37$

$\therefore x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{5 \pm \sqrt{37}}{6}$

didn't use correct formula.

i.e $\dfrac{b^2 \pm \sqrt{b - 4ac}}{2a}$

(b) Show clearly how the method of completing the square can be used to solve (2)

$$x^2 – 10x + 3 = 0$$

leaving the answer in exact form.

$x^2 - 10x = -3$

$x^2 - 10x + 25 = -3 + 25$

$(x - 5)^2 = 22$

$\therefore x - 5 = \pm\sqrt{22}$

$x = 5 \pm \sqrt{22}$

got to here but didn't solve for x

didn't look at formula sheet.

See Over

2. **[2 marks]**

The quadratic equation $kx^2 + kx + 7 = 0$ has exactly ONE solution. Find the value(s) of k.

$a = k, \ b = k, \ c = 7$

FOR 1 SOLUTION

$b^2 - 4ac = 0$

$\therefore k^2 - 28k = 0$ ✓

$k(k - 28) = 0$

$\therefore \underline{k = 0}$ or $\underline{k = 28}$ ✓

must reject or $0x^2 \therefore$ not quadratic

didn't use discriminant.

gave 2 sol'n not one.

3. **[1 marks]**

From the list below, CIRCLE those expression(s) that is/are polynomials.

★ $2x^5 - x^{0.5}$

$3x^3 - x^{-2} + 3x + 6$

$\boxed{\frac{2}{3}x^4 + 3x^2 - 4}$

$\boxed{(2x - 1)(3x + 5)(x^3 + 11)}$

$\sqrt{5x - 11}$

$\frac{1}{2x-3}$

★ comma error.

½ EACH.

4. **[1+1+1+1 = 4 marks]**

The time needed t hours to complete a journey by car is inversely proportional to the average speed v km/h. If the average speed of the car is 90 km/h then it takes 3 hours and 20 minutes to complete the journey.
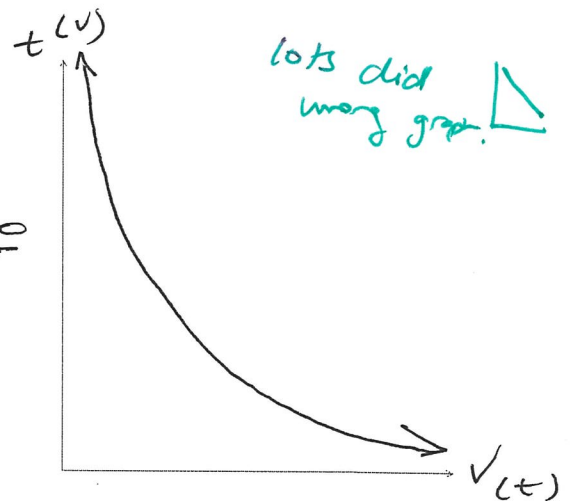
(a) Find the equation showing the relationship between t and v.

used minutes not hours.

$vt = 90 \times \frac{10}{3}$

$\therefore vt = 300$ ✓

or $t = \frac{300}{v}$

lots did wrong graph.

(b) How many km is the journey?

$\underline{300 \ km}$

(c) On the set of axes opposite, sketch a graph showing the relationship between t and v. ✓

(d) Hence or otherwise, find how long it will take to complete the journey at an average speed of 60 km/h

$t = \frac{300}{60}$ $\underline{t = 5 \ hours}$

See Over

/1

5. **[~~4~~ 5 marks]**

Solve $2x^3 + 5x^2 + x - 2 = 0$.

Let $P(x) = 2x^3 + 5x^2 + x - 2$

$P(-1) = -2 + 5 - 1 - 2$
$ = 0$

$\therefore x+1$ is a factor ✓

$(x+1)(ax^2 + bx + c) = 2x^3 + 5x^2 + x - 2$

$ax^3 = 2x^3$ $1 \cdot c = -2$ $(a+b)x = 5x^2$
$\underline{a = 2}$ $\underline{c = -2}$ $\therefore 2 + b = 5$ $\underline{b = 3}$ ✓

$\therefore (x+1)(2x^2 + 3x - 2) = 0$

$\Rightarrow (x+1)(2x - 1)(x + 2) = 0$ ✓

$\therefore x = -1 \; \wedge \; x = \frac{1}{2} \; \text{or} \; x = -2$

(margin, green) many stopped here

(right side, blue)
$2x^2 + 3x - 2$ `-4`
$2x^2 + 4x - x - 2$
$2x(x+2) - 1(x+2)$

(purple) didn't solve.

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6. **[6 marks]**

Consider the polynomial $P(x) = (2a - 1)x^4 + 9(b + 3)x^3 + 5x + \boxed{11 - c.}$

(a) What is the coefficient of the term involving x? $\underline{5}$ ✓   (1)

(b) What is the degree of P(x)? $\underline{4}$ ✓ *(green)* must say 4 not quartic or x^4 (1)

(c) Use the following information to find the values of a, b and c.   (3)

The leading term has a coefficient of 7, there is no constant term and the coefficient of x^3 is – 18.

$2a - 1 = 7$ $11 - c = 0$
$\therefore \underline{a = 4}$ ✓ $\therefore \underline{c = 11}$ ✓

$9(b+3) = -18$
$b + 3 = -2$
$\underline{b = -5}$ ✓

(green) assumed zero. not $11 - c = 0$.

(d) What would be the value of *a* if P(x) was to be monic?   (1)

$2a - 1 = 1$ $\therefore \underline{a = 1}$ ✓

(green) possibly guessed & got lucky $\frac{11}{11}$

See Over

7. **[5 marks]**

Draw a neat sketch of the graph of the function $y = 2x^3 + 7x^2 + 4x - 4 = (2x-1)(x+2)^2$.
Clearly label any significant points.

X-INT: $x = \frac{1}{2}, -2$

  $(\frac{1}{2}, 0)$ $(-2, 0)$
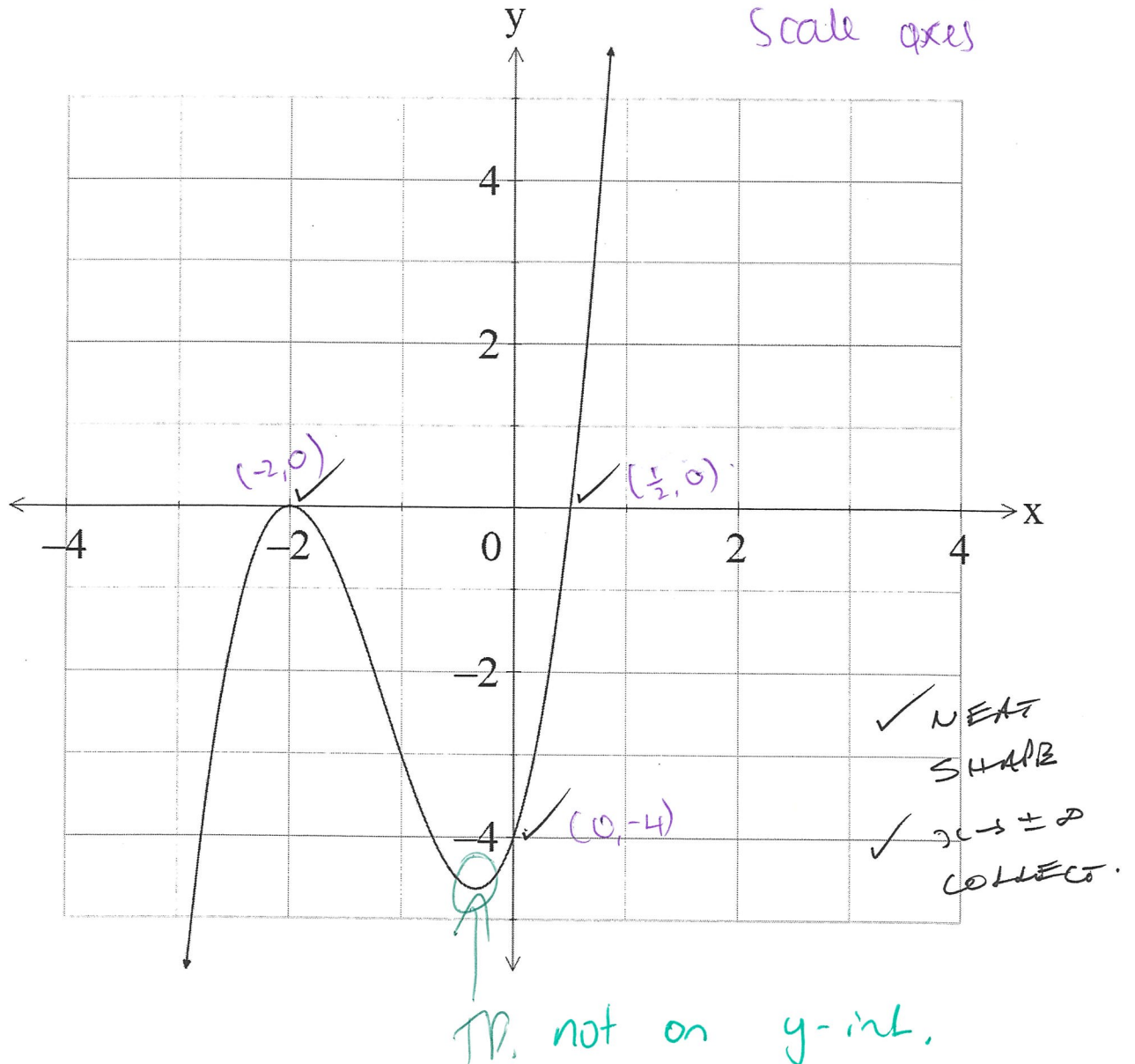
Y-INT: $y = (-1)(2)^2$

  $y = -4$

$x \to +\infty, \quad y \to +\infty$
$x \to -\infty, \quad y \to -\infty$ } not shown on graph.

make sure you scale axes

(-2,0) $(\frac{1}{2}, 0)$

(0,-4)

✓ NEAT
  SHAPE
✓ $x \to \pm \infty$
  COLLECT.

TP not on y-int.

End of Section One

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SENIOR HIGH SCHOOL

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~~24~~ 22

SECTION 2 – Resource Rich

Working Time: 20 minutes

8.   **[5 marks]**

The graph of a function y = f(x) is shown below.

(c)

ASYMPTOTES ✓

(c)

$\nearrow y = \frac{1}{x-3} - 4$
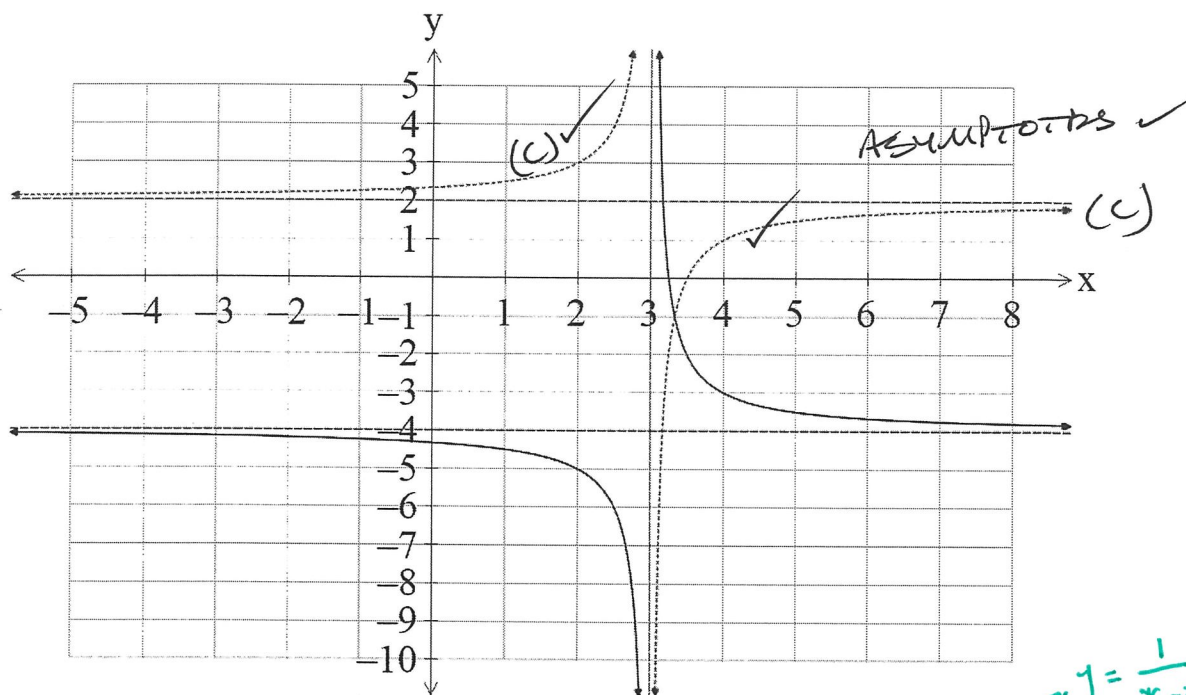
(a)   Write down the equations of the horizontal and vertical asymptotes.   (1)

$x = 3$, $y = -4$ ✓

gave eq'n not asymptoter

(b)   For what value(s) of x, if any, is f(x) = -4?   (1)

NONE ✓

(c)   Sketch y = - f(x) – 2 on the set of axes.   (3)

reflected accross y-axis

~~reflec~~ or. asymptots not x-axes.

See Over

/5

9. **[3 marks]** (marked "2" above, crossed through to 3)

A cubic equation has solutions x = -1, x = $1\frac{2}{3}$ and x = 4. Find the equation in the form $ax^3 + bx^2 + cx + d = 0$.

$$(x+1)(x-\tfrac{5}{3})(x-4) = 0 \checkmark$$

$$(x+1)(3x-5)(x-4) = 0$$
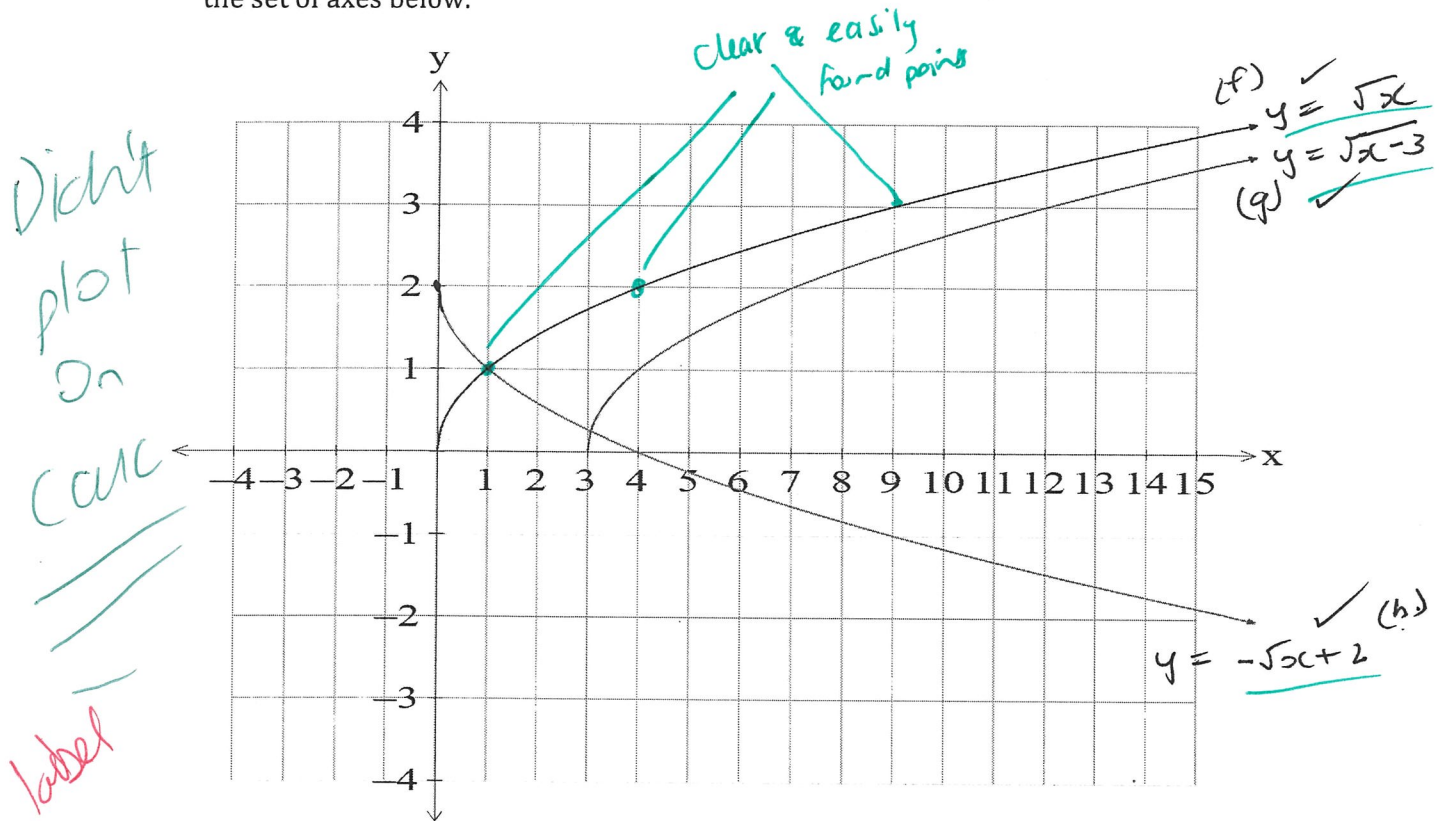
$$3x^3 - 16x^2 + 3x + 20 = 0 \checkmark$$

or

$$x^3 - \frac{14x}{3} + x + \frac{20}{3} = 0$$

10. **[6 marks]**

Consider the functions $f(x) = \sqrt{x}$, $g(x) = \sqrt{x-3}$ and $h(x) = -\sqrt{x}+2$.

(a) With the aid of your CLASSPAD, draw a neat sketch of the graph of each function on the set of axes below. (3)

Clear & easily found points

Didn't plot on calc

label

(f) $y = \sqrt{x}$ ✓
$y = \sqrt{x-3}$ (g) ✓

(h) $y = -\sqrt{x}+2$ ✓

(b) Describe how the graphs of f and g are related. (1)
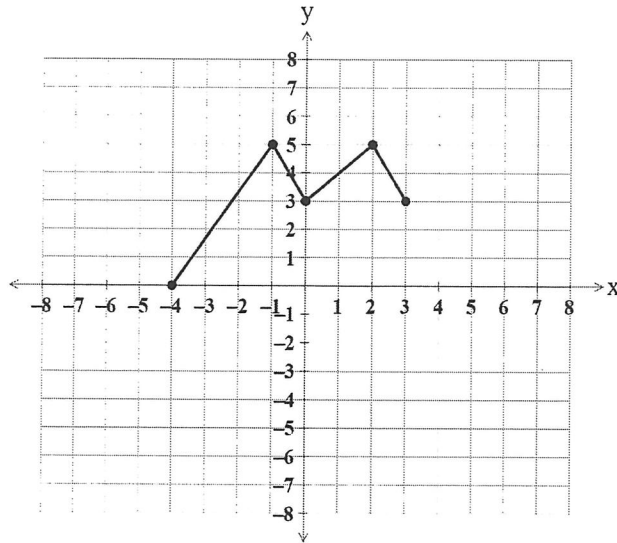
g is graph of f translated 3 units to the right ✓

(c) Describe how the graphs of f and h are related. (2)

many swapped order.

f is reflected in the x-axis ✓ then translated 2 units up.
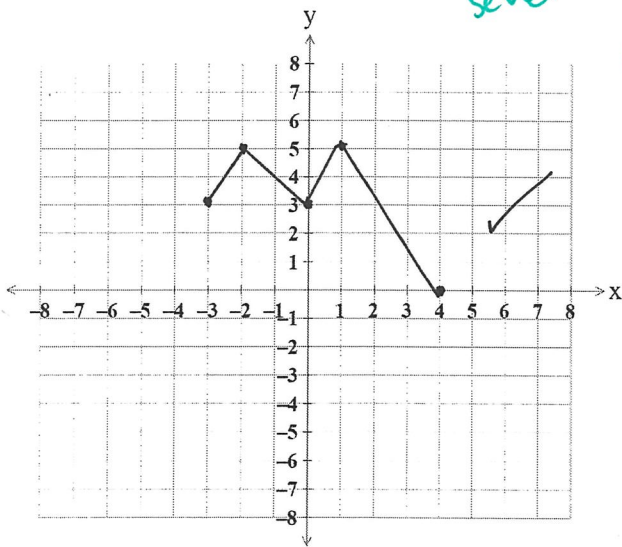
See Next Page

/8

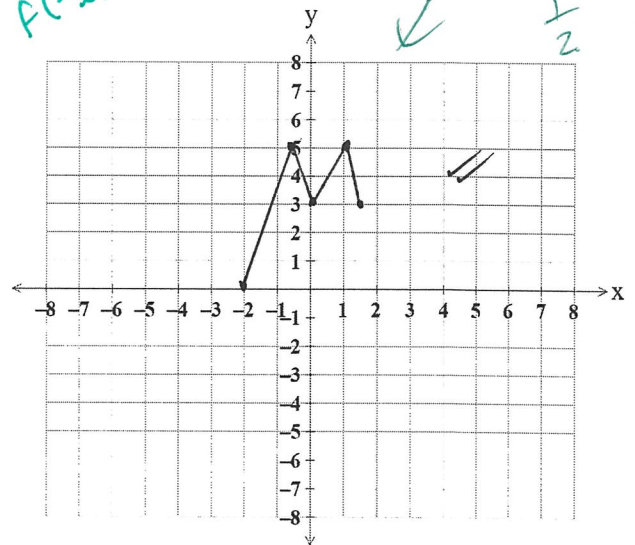**11.** [1+2+1 = 4 marks]

The graph of y = f(x) is shown below.

Draw the graph of each of the following.

y = f(-x)
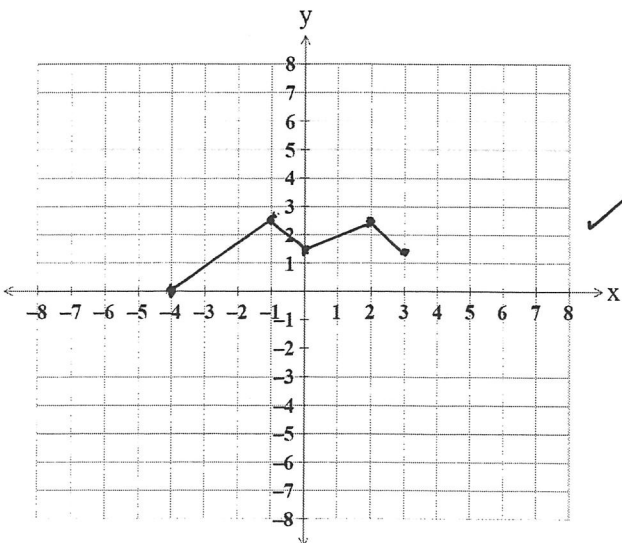

Several did subsequent transformations to f(-x) not f(x)

y = f(2x)

several students ← sf 2 not ½

y = 0.5f(x)

*many did vertical dilation of 2 not 0.5*

4

12. [1+2+1+1 = 5 marks]

Declan is trialling a new promotional strategy with the Facebook page for his small business. The number of "page likes" received each day since the start of his new campaign is found to be modelled by the following function, where L is the number of new "page likes" each day and x is the number or days since the commencement of the campaign.

$$L = \frac{x^3}{3} - 10x^2 + 64x + 25$$

put on calc!

A sketch of its graph is shown opposite.
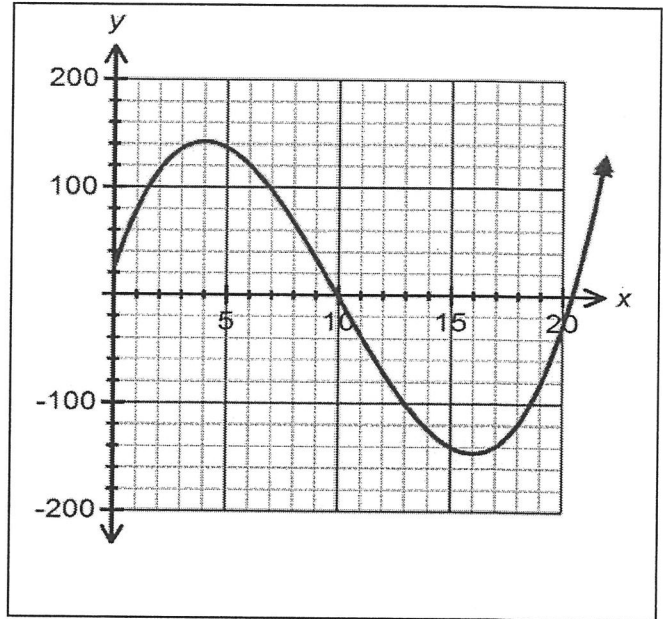
(a) Before the start of the strategy, how many new "likes" could Declan have expected to receive each day?

was y-int.

$\underline{25}$ ✓

ie $x=0$

(b) For how many days does the new strategy seem to have a positive effect after the start of the strategy?

above x-axis ≈ $\underline{9 \text{ days}}$ ✓ (9.9....)

ACCEPT. 10.

(c) At the most successful point in the strategy, how many new "likes" did Declan receive?

max on calc. ≈ $\underline{142}$ "Likes" (142.33....).

round

(d) For what value(s) of x does the function adequately model the situation?

$\underline{0 \leq x \leq 10}$ ✓ ✓

End of Section Two